

# A Traffic-pricing Model for the Packet-switching Network with Prioritized Round-robin Queueing<sup>1</sup>

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## Extended Abstract

Dynamic network traffic pricing has been considered a promising economic approach to solving network traffic congestion problem that has been diminishing the benefit of the Internet from e-businesses. This paper presents a traffic-pricing model for the packet-switching network that uses prioritized round-robin bandwidth allocation. We are exploring the scheme in which localized pricing can optimize global network performance. This research is aimed at industrial-application demands and intended to provide both the theoretical derivation and the practical scheme to the implementation of a traffic pricing system. We follow the methodology of stochastic general equilibrium used in the GSW model (Gupta, Stahl and Whinston, 1996, 1997) and focus the discussion on the service welfare optimization problem in elastic traffic control where the service delay is counted as the main constraint.

One of the important advantages of packet switching networks is the total delay a data flow, or alternatively a job, encounters from one host to another is less than the summation of the waiting and servicing time in all nodes along its path. This feature allows packet switching networks to provide better bandwidth efficiency than other types of network, such as circuit-switched networks. However, the “pipeline” effect in a packet switching network complicates

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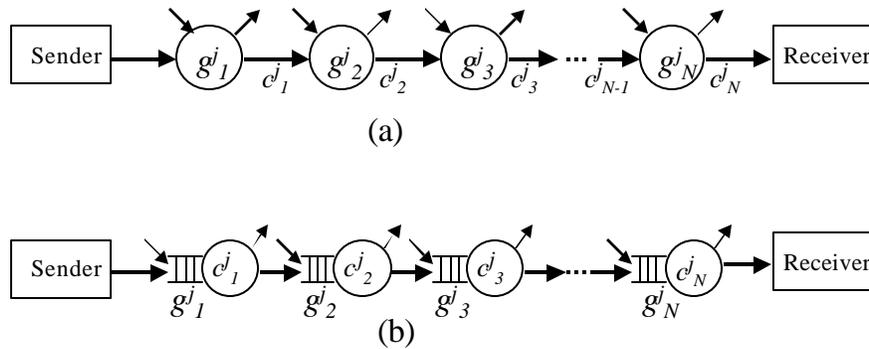
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traffic pricing processes because the total job throughput time is not a summation of throughput times of channels in a route.

A packet-switching network is defined as a set of distributed data transmission services enabling multiple routes simultaneously servicing different data flows. Structurally, a packet-switching network consists of a set of nodes and the channels between the nodes. A route forwarding a data flow from a sender to a receiver can be modeled as a series of queue service stations, each of which is made up of a queue in a network node and a bandwidth server by an outbound channel from the node. Assume that there is a packet switching network consisting of a set of  $G = \{g\}$  nodes. A channel  $c_s \in C$  is between two nodes, with service capacity featured as bandwidth  $B_s, s \in S$ . A route  $R^j$  transmits data flow  $j$  from a sender to a receiver (Figure (a)).  $R^j$  is a set of nodes and channels. It can be denoted as  $R^j = \{G^j, C^j\}$ , where  $G^j = \{g_1^j, g_2^j, \dots, g_n^j, \dots, g_N^j\} \in G$ ,  $C^j = \{c_1^j, c_2^j, \dots, c_n^j, \dots, c_N^j\} \in C$ , and  $N$  indicates the number of network nodes in the route and  $n = 1, 2, \dots, N$ .  $R^j$  can be modeled as a series of queueing systems. An individual queueing system in the series is made up of a queue in a network node and the bandwidth service by an outbound channel from the node (Figure (b)). When a channel  $c_n^j$  is busy, a queue forms at node  $g_n^j$ .



**Figure:** A route in a packet switching network is a series of queueing systems

We prove that the total throughput time a data flow is transmitted through a route can be expressed as the time spent at the bottleneck channel plus a relatively trivial amount of delay on other nodes of the path. Specifically, the residual transmission time is the time for a single data segment to go through all non-bottleneck nodes. This is because there is almost a total overlap in transmission time of the bottleneck channel and the other channels in the route. Since the size of a data segment is independent of the size of the data flow, the probability distribution of the extra throughput time on the remaining section of the path for a data flow is independent of the size of the data flow. The above "pipeline" effect of packet-switching networks adds complexity to traffic-pricing modeling.

To the packet-switching network with prioritized transmission services, we adopt a preemptive prioritization, i.e., a data flow joining the queue of the momentarily highest priority obtains the bandwidth immediately regardless of the service status of the data flows at lower priority classes. Data flows at the same priority share the bandwidth equally when the priority becomes the highest one among others.

A priced packet-switching network is an economy with data transmission services as the product and network users as the consumer. The prioritization results in differentiated product qualities with different levels of prices. The following economic conditions are assumed for establishing a traffic pricing mechanism:

- Data flows in the network, referred to as jobs, are inherent with values perceived by users as utilities;
- A job's net value to the user depends on two factors: the service price the user pays and the delay cost proportional to the throughput time;

- If jobs arrive at a channel momentarily in a rate that exceeds the channel's bandwidth they are queued;
- Prices are differentially set for the bandwidth of channels in accordance with service priorities and the traffic status of each priority;
- The information of both the bandwidth price and the expected throughput time are periodically disseminated to users;
- Users are rational, i.e., they will choose priority levels accordingly for their jobs to optimize the net benefit of the submission, and they submit their jobs only if expected net gains from job submissions are nonnegative.

There exist “natural demands”, i.e., maximum arrival rates of different types of jobs if there is no cost, to the network transmission service. However, the delay cost and the price paid for transmission services result in restricted job submissions. The objective of the traffic pricing is to dynamically choose a set of prices for the prioritized services at each network channel to suppress the traffic of lower valued jobs to a certain level. Thus, the submitted jobs result in an aggregate rate that turns out statistically the best service welfare rate. This issue can be formalized as a service-welfare rate maximization problem with benefit contributions from job submissions through various network routes made up of nodes and channels.

Solving the problem, we obtain two benefit-maximization unit prices for jobs of size  $q$  that are submitted to priority- $k$  service of channel  $s$ :

$$r_{qks}^* = \sum_{m \in J} \sum_{h \in K} \frac{\partial \Omega_{mhs}}{\partial \mathbf{j}_{qks}} y_{mh} \bar{\mathbf{d}}_m \text{ if channel } s \text{ is the bottleneck of the route, and}$$

$$r_{qks}' = \sum_{m \in J} \sum_{h \in K} \frac{1}{q_m} \frac{\partial \Omega_{mhs}}{\partial \mathbf{j}_{qks}} y_{mh} \bar{\mathbf{d}}_m \text{ if channel } s \text{ is not the bottleneck of the route,}$$

where  $J$  is the set of job types, and  $K$  is the set of priority classes,  $\bar{\mathbf{d}}_m$  is the submission rate weighted delay cost coefficient for type- $m$  jobs,  $y_{mh}$  is the arrival rate of type- $m$  jobs at priority- $h$  service,  $\mathbf{W}_{mhs}$  is the expected throughput time of type- $m$  jobs submitted to priority- $h$  service at channel  $s$ , and  $\mathbf{j}_{qks}$  is the arrival rate of jobs of size  $q$  submitted to priority- $k$  service at channel  $s$ .

Hence the necessary condition that the user submits a job can be expressed as:

$$V_{ij} - \mathbf{d}_{ij} T_{jk} \geq q_j (r_{jkH}^* + \sum_{s \in R^j, s \neq H} r_{jks}^*), \quad (*)$$

where  $V_{ij}$  is the gross value of type- $j$  job submitted by user  $i$ ,  $\mathbf{d}_{ij}$  is the delay cost coefficient for the job,  $T_{jk}$  is the expect total throughput time of the route for type- $j$  jobs submitted to the priority- $k$  service,  $q_j$  is job size,  $r_{jkH}^*$  is the bottleneck price for the priority- $k$  service of channel  $H$ , and  $r_{jks}^*$  is the non-bottleneck price for the priority- $k$  service of channel  $s$ . Channel  $H$  and other channel  $s$  are in the route  $R^j$  transporting the type- $j$  job.

Since the bottleneck price is much higher than prices for non-bottleneck channels, the criterion (\*) can be approximately expressed as

$$V_{ij} - \mathbf{d}_{ij} T_{jk} \geq q_j r_{jkH}^* \quad (**)$$

In this way, pricing for a route usage becomes for the bottleneck channel usage. This is obvious because the bottleneck channel is the dominant factor affecting the total throughput time of the route.

If there is more than one service class that meet (\*\*), the user will submit the job to a best service class

$$k^* = \sup_k \{ V_{ij} - \mathbf{d}_{ij} T_{jk} - q_j r_{jkH}^* \} \quad (***)$$

That is, the optimum service class maximizes the net value of the job.

The throughput time of type- $j$  jobs submitted to priority- $k$  service of channel  $s$  is  $W_{fks} = f(L_{ks}, q_j, B_{ks})$  depending on the queueing algorithm, where  $L_{ks}$  is the average number of jobs in priority- $k$ , and  $B_{ks}$  is the average bandwidth of channel  $s$  to priority  $k$  queue. We can obtain

$$L_{ks} = \frac{\mathbf{r}_{ks}}{1 - \sum_{h \leq k} \mathbf{r}_{hs}},$$

where  $\mathbf{r}_{ks}$  is the bandwidth utilization ratio of priority- $k$  service of channel  $s$ , and

$$B_{ks} = B_s (1 - \sum_{h < k} \mathbf{r}_h), \text{ where } B_s \text{ is the bandwidth of channel } s.$$

The appendix provides the derivation of the analytical form of the bandwidth-rental price for a single node situation, which can be easily generalized to multi-node route situations.

We applied the above traffic-pricing model to virtual private networks (VPNs) (Kosiur, 1998) to explore the implementation issues for a network traffic pricing system without prioritization (Lin, Ow, Stahl, and Whinston, 1999 and 2000). VPNs are referred to as private networks built upon a public network, specifically the Internet, to provide secured transmission services. VPN traffic pricing is both feasible and necessary because it is controllable to the organization operating the VPN, which is a social welfare maximizer. A VPN route consists of two channels between two VPN gateways and the Internet tunnel, and a virtual circuit in the Internet, which is made up of unknown number of channels between Internet routers. The Internet tunnel is not priced because it is assumed not the bottleneck in VPN data transmissions and is not controllable to the organization. Thus, the organization can build up a traffic pricing system for the VPN to optimize the bandwidth utilization of its Internet connections. The VPN job submission criterion is customized from (\*) as:

$$V_{ij} - \mathbf{d}_{ij} T_{jk} \geq q_j (r_{jKH}^* + r_{jKL}'),$$

where  $r_{jkH}^*$  is the unit price for the priority- $k$  service of the VPN-Internet bottleneck channel and  $r_{jKL}'$  is the unit price for the priority- $k$  service of another VPN-Internet channel that is not a bottleneck.

We further propose a *transaction-level pricing* architecture for the VPN traffic-pricing system to cope with issues such as digital contracting, VPN traffic pricing efficiency, logistic system construction, integration of the traffic-pricing system with existing traffic control techniques, and user acceptability. With transaction level implementation, the VPN allocates bandwidth requests on the per-transaction basis in regard to the application needs and priorities. We claim that this process provides the possibility to incorporate the pricing mechanism within enterprise resource planning applications.

We developed a prototype system named *VPN Traffic-Pricing Experiment System* (VTPES), which is built on a test network platform in the Center for Research in Electronic Commerce (CREC). VTPES is a scalable, distributed system operated with real-time data flows. Its dual-queue structure allows us to compare different experiment schemes using identical data traffic patterns. The experiments have showed the effectiveness of the pricing formulas derived from the traffic-pricing model. VTPES has been used for a series of diversified experiments, such as approximate job submission decision-making and network traffic pricing with human subjects. Currently, VTPES is being enhanced with the prioritized round-robin scheduling scheme. The outcomes are to be further reported.

## Appendix: The Optimum Pricing Formula for A Single Bandwidth Service with Prioritized Round-robin Queueing (PRR)

The prioritization of PRR is assumed preemptive. When there is no job in higher priority queues the round-robin algorithm is applied to each priority class. In this case, the available bandwidth to a priority class is independent to the status of the lower priority classes, but relevant to higher priority classes. Denote the service bandwidth as  $B$  and the set of priority classes as  $K$  with a smaller number for a higher priority. Then the available bandwidth to class- $l$  jobs is  $B$  and the available bandwidth to remaining classes is  $B(1 - \mathbf{r}_l)$ , where  $\mathbf{r}_k = \frac{\sum_{m \in J} \mathbf{j}_{mk} q_{mk}}{B}$  is a notation for the bandwidth utilization ratio of class- $k$  jobs. Since class-2 jobs always get service prior to other jobs at lower classes, their available bandwidth is  $B(1 - \mathbf{r}_1)$ . In general, the available bandwidth of class- $k$  jobs is

$$B_k = B \left( 1 - \sum_{h < k} \mathbf{r}_h \right).$$

According to Kleinrock (1975), the average number of jobs in a non-prioritized round-robin queue is:  $\frac{\mathbf{r}}{1 - \mathbf{r}}$ . This is applicable to the number of class- $l$  jobs in the case of prioritized

round-robin queueing. Denote  $\mathbf{r}'_k = \frac{\sum_{m \in J} \mathbf{j}_{mk} q_{mk}}{B_k} = \mathbf{r}_k \frac{B}{B_k}$ . In the similar way, the number of jobs

in a prioritized round-robin queue is:

$$L_k = \frac{\mathbf{r}'_k}{1 - \mathbf{r}'_k} = \frac{\mathbf{r}_k / (1 - \sum_{h < k} \mathbf{r}_h)}{1 - \mathbf{r}_k / (1 - \sum_{h < k} \mathbf{r}_h)} = \frac{\mathbf{r}_k}{1 - \sum_{h \leq k} \mathbf{r}_h}, \text{ for } k = 1, 2, \dots, K$$

The expected throughput time of type- $j$  job submitted to class- $k$  priority service with prioritized round-robin scheduling becomes:

$$\mathbf{t}_{jk} = \mathbf{W}_{jk}(L_k(\mathbf{q}, \mathbf{j}), q_{jk}, B) \gg L_k q_{jk} / B_k = \frac{\mathbf{r}_k q_{jk}}{B(1 - \sum_{h < k} \mathbf{r}_h)(1 - \sum_{h \leq k} \mathbf{r}_h)}, \text{ where } \mathbf{q} = \{q_{mh}\} \text{ and } \mathbf{j} =$$

$\{\mathbf{j}_{mh}\}$ , with  $m = 1, 2, \dots, J$  and  $h = 1, 2, \dots, K$ .

The first order condition turns out:

$$\frac{\partial \Omega_{jk}}{\partial \mathbf{j}_{mh}} = \frac{q_{mh} q_{jk} L_k^2 [2(1 - \sum_{l < k} \mathbf{r}_l) - \mathbf{r}_k]}{B^2 (1 - \sum_{l < k} \mathbf{r}_l)^2 \mathbf{r}_k}, \text{ if } h < k$$

$$\frac{\partial \Omega_{jk}}{\partial \mathbf{j}_{mh}} = \frac{q_{mk} q_{jk} L_k^2}{B^2 \mathbf{r}_k^2}, \text{ if } h = k$$

$$\frac{\partial \Omega_{jk}}{\partial \mathbf{j}_{mh}} = 0, \text{ if } h > k.$$

The price for type- $j$  job submitted to class- $k$  is

$$\begin{aligned} r_{jk} &= \sum_{l \in I} \sum_{m \in J} \sum_{h \in K} \frac{\partial \Omega_{mh}}{\partial \mathbf{j}_{jk}} x_{lmh} \bar{\mathbf{d}}_{lm} \\ &= \sum_{m \in J} \sum_{h \geq k} \frac{\partial \Omega_{mh}}{\partial \mathbf{j}_{jk}} \mathbf{j}_{mh} \bar{\mathbf{d}}_m \\ &= \sum_{m \in J} \sum_{h > k} \mathbf{j}_{mh} \bar{\mathbf{d}}_m \frac{q_{mh} q_{jk} L_k^2 [2(1 - \sum_{l < k} \mathbf{r}_l) - \mathbf{r}_k]}{B^2 (1 - \sum_{l < k} \mathbf{r}_l)^2 \mathbf{r}_k} + \sum_{m \in J} \mathbf{j}_{mh} \bar{\mathbf{d}}_m \frac{q_{mk} q_{jk} L_k^2}{B^2 \mathbf{r}_k^2} \\ &= \frac{q_{jk} L_k^2}{B^2 \mathbf{r}_k^2} \frac{2(1 - \sum_{l < k} \mathbf{r}_l) \mathbf{r}_k - \mathbf{r}_k^2 + (1 - \sum_{l < k} \mathbf{r}_l)^2}{(1 - \sum_{l < k} \mathbf{r}_l)^2} \sum_{m \in J} \sum_{h > k} q_{mk} \mathbf{j}_{mh} \bar{\mathbf{d}}_m \end{aligned}$$

Unit price

$$\begin{aligned}
r_k &= \frac{L_k^2}{B^2 r_k^2} \frac{2(1 - \sum_{l < k} r_l) r_k - r_k^2 + (1 - \sum_{l < k} r_l)^2}{(1 - \sum_{l < k} r_l)^2} \sum_{m \in J} \sum_{h > k} q_{mh} j_{mh} \bar{d}_m \\
&= \frac{L_k^2}{B r_k^2} \frac{(1 - \sum_{l \leq k} r_l)^2 - 2r_k^2}{(1 - \sum_{l < k} r_l)^2} r \bar{d}
\end{aligned}$$

where  $r = \sum_{k \in K} r_k$ ,  $\bar{d}$  is the flow rate weighted delay cost coefficient.

## References:

- Blake, S., D. Black, M. Carlson, E. Davies, Z. Wang, and W. Weiss, "An Architecture for Differentiated Services," Internet RFC2475, December 1998, <ftp://ftp.isi.edu/in-notes/rfc2475.txt>.
- Clark, D. and J. Wroclawski, "An Approach to Service Allocation in the Internet," INTERNET-DRAFT, July, 1997, <http://diffserv.lcs.mit.edu/Drafts/draft-clark-diff-svc-alloc-00.txt>
- Fishburn, Peter C., and Andrew M. Odlyzko, "Dynamic Behavior of Differential Pricing and Quality of Service Options for the Internet," *DIMACS Technical Report 99-12*, February 1999, <http://www.dimacs.rutgers.edu/TechnicalReports/>.
- Foukia, Noria et al, "User Behavior for a Pricing Scheme in a Multi-provider Scenario," *MIT/Tufts Internet Service Quality Economics Workshop*, December 2-3, 1999.
- Gibbens, R. and F. Kelley, "Resource pricing and the evolution of congestion control," *Automatica*, 1999, <http://www.statslab.cam.ac.uk/~frank/papers/evol.html>.
- Gupta, A., Stahl, D. and Whinston, A., "A Stochastic Equilibrium Model of Internet Pricing," *Journal of Economic Dynamics and Control*, 21, 1997(a), pp697-722.
- Gupta, A., Stahl, D. and Whinston, A., "Economic Issues in Electronic Commerce", Chapter 8 in *Readings in Electronic Commerce*, Kalakota, R., and A.B. Whinston eds., Addison-Wesley, Massachusetts, 1997(b).
- Gupta, A., Stahl, D. and Whinston, A., "A Priority Pricing Approach to Manage Multi-Service Class Networks in Real-Time," L. McKnight and J. P. Bailey eds., *Internet Economics*, MIT Press, Cambridge, 1998.
- Gupta, Alok, Dale O. Stahl and Andrew B. Whinston, "The Economics of Network Management," *Communications of the ACM*, (42:9), September 1999, pp57-63.

- Jacobson, Van and Michael J. Karels, "Congestion Avoidance and Control," *SIGCOMM'88*, ACM, 1988.
- Kalyanaroman, S., T. Ravichandran and Randy Norsworthy, "Dynamic Capacity Contracting: A Framework for Pricing the Differentiated Services Internet," May 1998, <http://www.ecse.rpi.edu/Homepages/shivkuma/research/papers/icepaper.html>.
- Kleinrock, Leonard, *Queueing Systems*, Vol. I & II, John Wiley & Sons, 1975.
- Kosiur, Dave, "Building and Managing Virtual Private Networks," John Wiley & Sons, 1998.
- Lin, Zhangxi, Peng Si Ow, Dale O. Stahl, and Andrew B. Whinston, "Exploring Traffic Pricing for the Virtual Private Network," WITS'99, December 11-12, 1999, Charlotte, North Carolina.
- Lin, Zhangxi, Peng Si Ow, Dale O. Stahl, and Andrew B. Whinston, "Exploring Traffic Pricing for the Virtual Private Network," under review by *Information Technology and Management*, May, 2000.
- MacKie-Mason, J., and H. Varian, "Pricing the Internet." *In Public Access to the Internet*, ed. B. Kahin and J. Keller. Englewood Cliffs, NJ:Prentice-Hall, 1995.
- Odlyzko, Andrew M., "A Modest Proposal for Preventing Internet Congestion," *DIMACS Technical Report 97-68*, October 1997, <http://www.dimacs.rutgers.edu/TechnicalReports/>.
- Shenker, Scott, David Clark, Deborah Estrin and Shai Herzog, "Pricing in Computer Networks: Reshaping the Research Agenda," *Telecommunication Policy*, (20:3), pp183-201, 1996.
- Stevens, W., "TCP Slow Start, Congestion Avoidance, Fast Retransmit, and Fast Recovery Algorithms," Internet RFC2001, January 1997, <http://info.internet.isi.edu/in-notes/rfc/files/rfc2001.txt>.
- Varian, Hal R., "Estimating the Demand for Bandwidth," *MIT/Tufts Internet Service Quality Economics Workshop*, December 2-3, 1999.